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# PHENOMENOLOGICAL MODELING OF SOUND FIELD GENERATED BY TRANSVERSAL VIBRATIONS OF STRAIT-LINED AND CYLINDRICAL SOURCES

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**ABSTRACT.** The paper presents two physical models meant to calculate the characteristics of noise fields produced by real strait-lined sources. Our study highlights the complexity of phenomenological and mathematical correlations between the parameters that should depict the field of traveling waves generated by real cylindrical surfaces. Acoustically, the distribution of sound flows is modeled through beady vibrating string or cylindrical-tube sources. Our results, in conjunction with sound level sonometric measurements, permit many comparative analysis of effective perturbation in sound propagation. Together with point or spherical models, our results can be implemented in the training of future specialists in applied acoustics of environmental ecology.

***Keywords:*** *Strait-lined and cylindrical sources, noise level distribution modeling*

## 1. INTRODUCTION

The vibrating systems become sources of simpler or more complex sounds, depending on how the normal modes are excited, more or less relevantly. These facts are most obvious on the vibrating strings, plates, blades and tubes. Through pulsed collision, their walls transmit momentum to the molecular layer of ambient air and so compressional waves that travel by successive compressions and relaxations are generated. Physiologically, these ondulatory or wave movements, also called sounds, are perceptible by the sense of hearing. Waves behaviors are determined by the inertial and elastic properties of air, through the density ρ, and the parameter $\gamma p_0$

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(γ - adiabatic exponent, $p_0$-atmospheric pressure). Each mode of oscillation, from the set of possible modes of vibrating sources, acts as a simple oscillator in a forced sustained oscillation. These sources transmit a part of their energy to the air and thus the system will be maintained stationary. As the emission of successive wave fronts is normal, they will also have a geometric shape similar to the radiating surface.

Due to restoring forces in a local environment, disturbances occur and they spread almost as mechanical waves. This mechanism and progressive form of vibratory motion can be described by traveling oscillatory displacements and changes of local pressure. Acoustically, in real cases, the noise field should be taken as those generated by stationary incoherent, extended sources, or by judicious distributed sources [1, 2, 3]. For calculation the spatial distribution of noise intensity, one has to define the principles which justify the connection with the energetic-stochastic character of the real composition of the sound field generated by the equivalent sources. So, our models relate to the study of noise levels produced at the receptors placed in different points of sound field.

## 2. MODELING OF SOUND FIELDS GENERATED BY STRAIT-LINED SOURCES

Dynamically, the amounts of pressure and intensity of sound wave will characterize, in time and in space, this very widespread and complex form of undulatory motion of matter. Considering the field of restoring forces as a conservative one, the rate of energy exchanges into and out of a given region of the fluid will be equal. The spreading mechanism of the deformation and movement is specified by the instant, local mater density and intensity of the wave's energy. The relationship between them shows that waves carry mechanical vibratory energy through elastic medium without an effective drift-movement of the mater, or that all waves, mechanical and of other nature, have a macroscopic traveling-progressive character.

All possible vibration modes of the tubular sources will be assimilated to a breathing mode in which the cylinder wall radius expands and contracts. All parts are moving at the same rate. This type of vibrators, called monopole, radiates sound equally [4, 5] well in all directions. The sound pressure radiated by a monopole source varies with angle, but pressure is the same for all radial angles. This moving shows the motion of the cylinder as well as the resulting motion of the particles in the fluid surrounding the circle cylinder. Each particle oscillates back and forth about some equilibrium radius. Thus, the phase of their relative motions produces an outward traveling spherical wave. In a non-dispersive medium the wave-fronts (points of constant phase) are circles and the maximum particle displacement (amplitude) is the same at any point on a wave-front. According to superposition principle, the study of incoherent waves emitted by real sound radiators implicates the use the energetic scalar amounts. Thus, in each considered volume of the sound field, the integral energy of the complex wave will result as a sum of all specific energies of each component mode of complex generative vibration.

Considering the sound intensity as the rate of wave's energy passing through the unit of oriented area ***S,*** around a point on the wave front [6, 7, 8], the local density of wave energy, related to sound intensity, will respect the equations:

$$\varepsilon = \frac{\delta E}{\delta V} = \frac{W\delta t}{\mathrm{S v}\delta t} = \frac{W}{S\mathrm{v}} = \frac{I}{\mathrm{v}} \quad \Rightarrow \vec{I} = \frac{W}{\vec{S}} = \varepsilon\,\vec{\mathrm{v}} \tag{1}$$

where *W* is the power of sound flow. Trough wave front speed, **v,** we can see the vectorial character of the wave intensity ***I***. Based on the above considerations we developed two models for strait-linear noise sources.

### 2.1. Model of rectilinear beady sources

To develop the model for calculating the intensity of noise produced by rectilinear source, we consider that this is equivalent with a strait-lined string system consisting of punctual or spherical adjacent beady sources, emitting each omni-directionally. In figure 1, one can follow all the considerations for finding the sound intensity in the observation point **P** located at the distance *r* from the axis Ox of the rectilinear source.

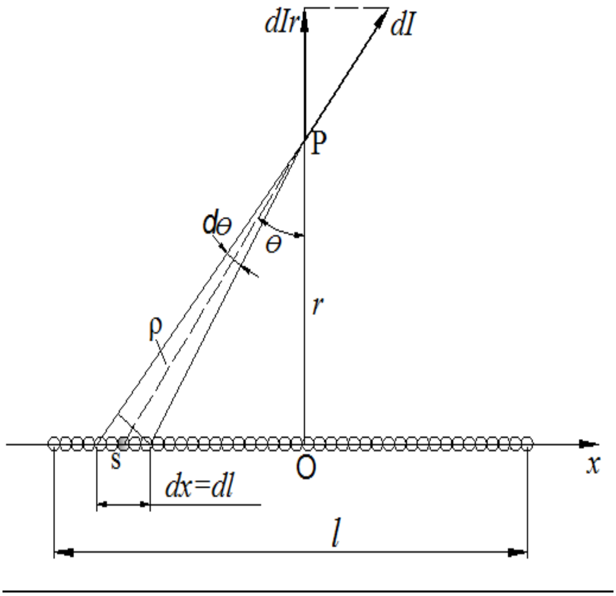


**Fig. 1**. Linear beady sound source

Considering the infinitesimal length *dl=dx* of vibrating rectilinear beady sources, the linear density of emission being *w = dW/dl*, as in the case of a point or spherical source, its contribution to the intensity of the sound field at the point P, will be:

$$dI = \frac{wdl}{4\pi\rho^2},$$

Consistently, the radial intensity projection normally to the sources direction is:

$$dI_r = \frac{wdx}{4\pi\rho^2}\cos\theta$$

Noting that we have the distances *ρ=r/cosϑ* and *dl=dx=ρdϑ/cosϑ* and then substituting, we obtain the radial contribution to the intensity of the punctual source *S*, of coordinate *x*=OS.

Considering a sufficiently long rectilinear beady string sources (comparatively with the observation distance) the angle *ϑ* takes values ranging from -π / 2 to + π / 2. Thus, the full amount of contributions to the intensity will be:

$$I_r = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{w\cos\theta}{4\pi r} d\theta = \frac{w\sin\theta}{4\pi r}\Bigg|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} == \frac{w}{4\pi r}(1+1) = \frac{w}{2\pi r} \qquad (2)$$

So, the received noises inverse-proportionally decrease with the distance to strait-lined source. For a real flow consisting of noise sources having diverse emission powers and forms, the linear density of the emission can be considered as the statistical average for each particular w=w(r) distribution.

**2. 2. The model of tubular sound source** (Hollow cylinder sound source model).

We present a phenomenological model of tubular sound sources that explain more intuitive the emission (radiation ) of the sound. This is based on the normal transfer of momentum and mechanical energy from the wall transversal vibration of hollow-cylinder to the air which becomes the porter of progressive compressional (longitudinal) sound waves.

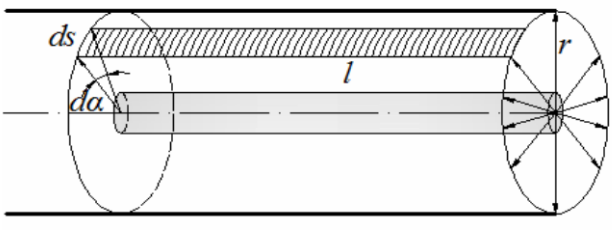


**Fig. 2.** Hollow cylinder sound source

For finding the spatial distribution of intensity in the fields generated by strait-lined sources, we will take into account that a cylindrical-tubular source radiates wave field with a radial symmetry in two dimensions. The transverse vibrations of the hollow-cylinder walls act upon air strata generating radial oscillatory forces and thereby compressional wave fronts will propagate in the form of coaxial cylindrical layers.

In figure 2 one can see that the infinitesimal area of a thin strip included between two generatrix of the cylindrical wave front will be *dS=lds=lrdα*, where *l* is the length, *ds=rdα* the strip width and *dα* is the plan angle under which is seen the arc *ds* located at distance *r* from the axis of the tubular source. Under these conditions, the area through which the sound emitted flows will be:

$$S = \int_0^{2\pi} l r d\alpha = 2\pi r l$$

So, in a point of the sound field generated by the cylindrical source with radial emission power *W*, the noise intensity will be:

$$I(r) = \frac{E}{tS} = \frac{W}{2\pi rl} = \frac{wl}{2\pi r l} = \frac{w}{2\pi r} \qquad (2')$$

where *w=W/l* is the power output per unit length of the source. The physical limitation $r \geq r_0$ is required, i.e. the distances to observer have to be equal or greater than the radius $r_0$ of the hollow-cylindrical source.

It is noted that these two modeling, by pearly spherical string and respectively hollow-cylindrical sources, lead to the same sound intensity expressions (2) and (2'). This may indicate that the stationary vibrations of these sources generate a sound field with cylindrical symmetry, which topographically has a wave-form of coaxial cylindrical layers.

Because the sound intensity decreases inverse proportionally with the distance from the axial source and between two cylindrical coaxial surfaces of rays $r_1$ and $r_2$ the radial waves system carries the same energy per unit time, we have:

$$\frac{I_1}{I_2} = \frac{r_2}{r_1} \text{ and the pressure } \frac{p_1}{p_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{r_2}{r_1}}$$

As the sound pressure is proportional to the oscillation amplitude and to the speed of these movements, it is evident that the elongation of the two-dimensional waves decreases inverse proportionally to the square root of the distance. So, with $r^2=x^2+y^2$, it is assumed that the kinematics equation of the undulatory sinusoidal movement will be:

$$\Psi(r,t) = \frac{A(0)}{\sqrt{r}} \sin \tilde{\omega}(t - \frac{r}{\mathrm{v}}) \qquad (3)$$

This function can be considered a consistent solution of the wave's differential equation in two-dimensions:

$$\frac{\partial^2\Psi}{\partial t^2} = \frac{v^2}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\Psi}{\partial r}$$

Thus, the modeling of the sound intensity generated by linear beady or tubular cylinder sources is confirming through cylindrical symmetry of elongations Ψ(*r, t* ) of 2D radial scattering.

Some sources of sound such as e.g. the vibrating string of musical instruments can be, mathematically, approximated by a (circular) cylindrical source of infinite extent, which facilitates an analytical solution. [9, 10].

## 3. LEVEL OF NOISE GENERATED BY THE STRAIT-LINED SOURCES

The loudness of psychophysical normal audible sensations, including other noxious effects, increases logarithmically with the increasing of physical stimuli intensity. For characterize auditory sensation, it is unanimous accepted to use so called noise level that is the logarithm of relative intensity of sounds. It is a dimensionless quantity that measures, less subjectively, the energy action effect of the stimulus upon sensory organs that, psychologically, is reflected through the loudness of sensation [11, 12].

Using equations (2) the sound level at the distance *r* from a linear-beady or hollow-cylindrical source, having the emission power per unit length *w,* will be:

$$L(r) = 10\log\frac{I(r)}{I_{ref}} = 10\log\frac{w}{2\pi r I_{ref}} = 10\log\frac{w}{10^{-12}} - 10\log r - 10\log 2\pi \Rightarrow \quad (4)$$

$$L = L_w - 10\log r - 8$$

The dimensional homogeneity of these equations is ensured if: $w_{ref}=10^{-12}$ $W/m$ and $r_{ref}=1m$. $L_w$=120log*w* is the level of sound power per length unit of source with cylindrical symmetric emission.

Usually, the sound loudness or listening intensity in a point is evaluated by the level of sound pressure. Since $I=p^2/\rho v$ with $I_{ref}=10^{-12}$ W·m$^{-2}$ and reference pressure $p_{ref}=2\cdot10^{-5}$ Pa, between the sound level referred to pressure and to intensity in a given point of the sound field, under normal conditions of air, there is a difference of less than one decibel, witch usually, is neglected. Indeed:

$$10\log\left(\frac{p^2(r)}{p_{ref}{}^2}\frac{I_{ref}}{I(r)}\right) = 10\log\left(\frac{\rho \mathrm{v} I_{ref}}{p_{ref}{}^2}\right) = 10\log\frac{1.28\cdot 331\cdot 10^{-12}}{4\cdot 10^{-10}} = 0.25\,dB \quad (5)$$

According to equations (5), through divergent cylindrical symmetry scattering, the level of the noise generated by strait-lined sources decreases linearly with the logarithm of the distance to axis source, slower than those with spherical scattering, which decreases with log$r^2$.

In reality, due to absorption, the amplitude of progressive wave is exponentially attenuated with distance. For the waves with cylindrical symmetry, the amplitude is given by the equation:

$$A(r) = \frac{A(0)}{\sqrt{r}} e^{-\chi r}$$

where *A(0)= A(r=R)* is the amplitude of sound waves excited in the air at the surface of the hollow-cylindrical source. The amplitude attenuation coefficient $\chi$ is one half of the intensity absorption coefficient ($\alpha=2\chi$) noise level in air will be:

$$L\ (r) = L_w - 10\alpha|r| - 10\log|r| - 8 = L_{W_0} - 10(\chi\, r + \log r) - 8 \qquad (6)$$

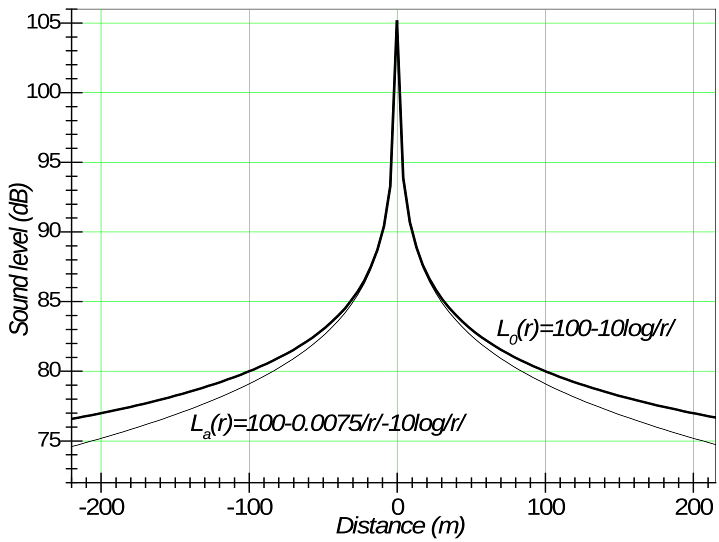


**Fig 3.** Level decreasing of noise field generated by strait-lined source.

In figure 3 graphs one can see the variation of the real level $L_{a(r)}$ comparatively with theoretical level $L_0(r)$, without absorption, only through cylindrical divergent scattering. It is necessary to note that in habitual case of air, the absorption causes a linear decreasing of the noise level stronger than the divergent scattering.

## 4. CONCLUSIONS

Starting from the physical-energetic analysis, we have modeled the emission and propagation of noises produced by rectilinear sources through transversal vibrations of hollow-cylinder or beady string. Our study on modeling the noise field generated by strait-line sources highlights the complexity of phenomenological and mathematical correlations between the observables that characterize sound fields. Having a high degree of generality, our models permit a phenomenological understanding and study of the sound intensity spatial distribution and of energetic and acoustic noise parameters. According to both models, through divergent cylindrical symmetry scattering, the level of the noise generated by strait-lined source decreases linearly with the logarithm of distance to source's axis ($\log r$), slower than for spherical sources with three dimensions sound scattering, which decreases with $\log r^2$. So through scattering, in the noise field, the power dissipation is a 3D phenomenon if the sound front expands as the surface of a sphere $4\pi r^2$ and it is a 2D phenomenon if sound front expands in two dimensions as the circumference $2\pi r$ of a circle.

Correlated with the acoustic and geometric parameters of composed strait-lined sources, trough stochastic mean values, our modeling also allows a prediction of sound level and of noise exposure dose. In conjunction with sonometric measurements and spectral noises analysis, our models can be used for analysis of other various facts of environmental and industrial acoustics. The results presented in this paper may have a wide field of applications and developments. Much of acoustic concepts and data presented in [13, 14, 15, 16], phenomenological explained and specified in our work, can be also implemented in the training of future specialists in environmental ecology.

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