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OPTIMUM LEVEL OF INVESTMENT IN EDUCATION: SOME LESSONS FROM AN ENDOGENOUS GROWTH MODEL

Ioana VIAȘU*
West University of Timisoara, Romania

Abstract. Previous results show that, at equilibrium, the common growth rate is independent of the share of resources spent on education. This paper, not only improves and extends these results, but also justifies some other approaches developed. It develops new results concerning the relationship existing between the economic growth and the resource allocated to education by assuming the case of a two sectors endogenous growth model, with the hypothesis that the share of resources spent on education is a control variable. This hypothesis is in perfect accordance with the economic reality. The share of resources spent on education is chosen by governments or by individuals and thus, this quantity cannot be arbitrarily chosen. As a consequence, it has to be a control variable in an optimal program. In this way, the share of resources spent on education, determined from the optimal problem, coincides almost exactly with that of developed countries.

JEL Classification: C61, J22, O41

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1. Introduction

The educational sector plays a crucial role in the process of the creation of human skills, which are the essential element of human capital. Almost all countries with high level of economic growth have labor forces with a high level of education. That is why, as it was pointed out by Lucas (1988), investment in education contributes to economic growth just as investments in the physical capital do. There is a considerable literature on this subject, both at a theoretical and empirical level. The list is extremely large and beyond the scope of the present paper. However we mention here only some papers, the most important on this field, as those of Benhabib and Spiegel (1994), Judson (1998), Mauro and Carmeci (2003), Vandenbussche et al. (2006), Hanushek and Kimko (2000), Hanushek and Woessmann (2008) and Aghion et al. (2009). Some of these papers are theoretical studies, trying to clarify the relationship between human capital and economic growth. Also, a considerable number of papers are empirical studies that confirm the strong impact of education on economic growth. Education plays a critical role in creating

* Corresponding author. Address: Faculty of Economics and Business Administration, West University of Timisoara, Pestalozzi, 16, 300115, Timişoara, Romania., Tel. +40.256 59 25 06, Email: ioana.viasu@e-uvt.ro

human capital, which contributes to production and economic growth just as physical capital and labor do. Almost all countries with high level of economic growth have labor forces with high level of education.

Vandenbussche et al. (2006) claim that education will increase the innovative capacity of the economy creating new knowledge and new technologies that generate growth. In fact, education adds new skills to labor and increases the capacity of labor to produce more output. However, some empirical contributions (Benhabib and Spiegel, 1994) find a positive effect on output growth of the stock of human capital and not of human capital accumulation. Other studies on the determinants of economic growth (Hanushek and Kimko, 2000, Hanushek and Woessmann, 2008) find a statistically significant positive effect of the quality of education on economic growth.

An interesting approach is presented in the paper of Aghion et al. (2009). They suggest that the relatively slow growth rate in European Union can be explained by the under-investment in higher education. The European Union invested only 1.1% of its total annual gross domestic product in higher education compared with 3% in the United States. Aghion et al. (2009) give some answers, concentrating their analysis on the economy of the United States, by means of two models - one with migration and the other without migration. A similar procedure is presented in the paper of Judson (1998), where she introduces a measure of efficiency's allocations of educational resources. The conclusion is that countries whose allocations are inefficient by this measure are gaining little from their investments in education: compared to countries whose allocations are more efficient. This finding has important implications for investment in education: if countries want to spur growth through investment in human capital, they must not invest indiscriminately. Of course, we can admit that the measure of efficiency introduced by Judson can be an argument to consider the share of resources spent on education as a control variable, but only a preliminary argument. We shall explain later the true argument and as we can see, it is connected to the neoclassic theory.

Mauro and Carmeci (2003) propose a model of endogenous growth with inefficiencies in the production of human capital caused by unemployment. Their model implies a negative long run relationship between growth and equilibrium unemployment. As was indicated by both authors, in spite of the previous results in the literature, if we can control the rate of unemployment, then the human capital accumulation has positive effects in the long run on economic growth.

In a recent paper, Chilarescu and Viasu (2014) develop a new model of endogenous growth, in which the dynamics of human capital is determined, not only by the stock of human capital and by the percentage of time dedicated to schooling, but also by the share of resources spent on education. However, in the model developed by Chilarescu and Viasu (2014), the growth rate is not affected by the amount of resources invested in education. In order to improve these results another model of endogenous growth is developed here. This time, the share of resources spent on education is a control variable. This approach is not just a simple alternative to the model developed in the cited paper, but more than this. As it is well-known, the share of resources spent on education is chosen by governments or by individuals and this quantity cannot be arbitrarily chosen. Of course, different alternatives are conceivable, but the only one that corresponds to the neoclassical theory is that where the choice is made in an optimal way. That is why this paper considers that the share of resources spent on education must be a control variable in an optimal program.

This paper is organized as follows. The second section contains the theoretical model, the third section determines the equations describing the optimal path, the fourth section studies the balanced growth path and gives some numerical simulations and the final section contains conclusions and reflections on our results.

2. The research model

This section recapitulates the Chilarescu and Viasu (2014) model, assuming a different function for the accumulation of human capital, which takes into account the amount of resources allocated to education. That is why we consider the same case of a two-sector growth model, where the first one is the goods sector that produces physical capital and the second sector is the education sector that produces human capital, both of them under conditions of constant returns to scale. The output in the good sector is produced using a Cobb-Douglas technology. Without loss of generality, we suppose that the economy is populated by a large and constant number of identical individuals, normalized to one, so that all the variables can be interpreted as per capita quantities.

$$y(t) = f[k(t), h(t), u(t)] = A[k(t)]^{\beta}[h(t)u(t)]^{1-\beta}, \beta \in (0,1) \qquad (1),$$

where $k(t)$ is physical capital, $h(t)$ is human capital that means the individual skill level, assumed to be identical for all persons employed and $u(t)$ is the fraction of time-labor allocated to the production of physical capital. β is the elasticity of output with respect to physical capital and A is a positive technology parameter. The equation describing the resources constraints of the economy is:

$$y(t) = I_e(t) + I_k(t) + c(t), \quad (2)$$

where $I_e(t)$ means investment in education, $I_k(t)$ means investment in physical capital and $c(t)$ means per-capita consumption.

As it is well known, one of the main purposes of the resources invested in education is to increase human skills that mean human capital. The amount of the resources invested in education will be chosen by the individual or by the government. In contrast to the paper of Chilarescu and Viasu, we suppose here that $I_e(t)$ is a variable percentage of total output, that is $I_e(t) = \pi(t)y(t)$.

Naturally these variables are all functions of time, but when no confusion is possible, we write simply y, k, h, π and u. Substituting $I_e = \pi y$ into the resource equation (2) we can write following standard differential equation that describes the dynamics of physical capital:

$$\dot{k} = (1-\pi)Ak^{\beta}(hu)^{1-\beta} - c. \qquad (3)$$

Differently from the original model of Lucas I consider that the dynamics of human capital is determined not only by the fraction of time devoted to education, but also by the amount of resources allocated to education, and can be described by the following differential equation:

$$\dot{h} = \delta\pi(1-u)Ak^{\beta}(hu)^{1-\beta}, \qquad \delta > 0, \qquad (4)$$

where $\delta > 0$ is the efficiency parameter of the educational sector. As a consequence of this assumption, we can observe that if $\pi > 0$, then each individual can allocate a fraction $1-u$ of his budget of time to the educational sector and if $\pi = 0$, then each individual will allocate his budget of time only to the production sector, that means $u = 1$.

Accordingly, to the above formulations, the standard utility function is not applicable. We cannot assume that consumers' utility, at instant t, depends only on consumption at instant t. Simply because the utility function will be affected by the amount invested in education, of course, with future positive consequences. What we need is an utility function that is not separable in consumption and share of resources spent on education. For more details see the papers of Alonso-Carrera et al. (2005) and Pintus (2007). Consequently, the instantaneous utility function that we propose takes the following form:

$$U(c,\pi) = \frac{c^{1-\theta}(1-\pi)^{\gamma(1-\theta)} - 1}{1-\theta}, for\ all\ \theta > 0, \theta \neq 1\ and\ \gamma\ \in (0,1). \qquad (5)$$

γ and θ are constant parameters. γ tries to attenuate the effect of investment in education on current consumption, and θ is the elasticity of marginal utility with respect to consumption and coincides with the inverse of the constant elasticity of inter temporal substitution when γ = 0. When θ tends to one, it not difficult to show that

$$U(c,\pi) = \ln(c) + \gamma \ln(1-\pi).$$

This utility function is inspired by the papers of Alonso-Carrera et al. (2005), Bennett and Farmer (2000), Carroll et al. (1997) and Pintus (2007). The instantaneous utility function is obviously a concave function for all $\theta > \frac{1}{1+\gamma}$, increasing in consumption and decreasing in share of resources invested in education.

3. The optimal path

Without loss of generality we assume that A = 1 and thus the optimization problem can be written as follows.

Definition 1. *The set of paths {k, h, c, u, π} is called an optimal solution if it solves the following optimization problem:*

$$V_0 = \max_{u,c,\pi} \int_0^\infty \frac{c^{1-\theta}(1-\pi)^{\gamma(1-\theta)} - 1}{1-\theta} e^{-\rho t} dt, \qquad (6)$$

subject to

$$\begin{cases} \dot{k} = (1-\pi)Ak^\beta (hu)^{1-\beta} - c \\ \dot{h} = \delta\pi(1-u)Ak^\beta (hu)^{1-\beta} \\ k_0 = k(0) > 0,\ h_0 = h(0) > 0. \end{cases} \qquad (7)$$

The system (7) gives the resources constraints and initial values for the state variables *k* and *h*. To solve the problem (6) subject to (7), we define the Hamiltonian function:

$$H = \frac{c^{1-\theta}(1-\pi)^{\gamma(1-\theta)} - 1}{1-\theta} + \left[(1-\pi)Ak^\beta (hu)^{1-\beta} - c\right]\lambda + \delta\pi(1-u)Ak^\beta (hu)^{1-\beta}\mu.$$

The boundary conditions include initial values for human and physical capital and the transversality conditions:

$$\lim_{t\to\infty} \lambda(t)k(t)e^{-\rho t} = 0, \qquad (8)$$

$$\lim_{t\to\infty} \mu(t)h(t)e^{-\rho t} = 0. \qquad (9)$$

In this model, there are three control variables, *c*, *u* and π and two state variables, *k* and *h*. In an optimal program the control variables are chosen so as to maximize *H*. We note that along the optimal path, λ and μ are functions of *t* only. The necessary first order conditions for the (*c*, π, *u*) to be an optimal control are:

$$\begin{cases} \dfrac{\partial H}{\partial c} = 0 \Rightarrow \lambda = c^{-\theta}(1-\pi)^{\gamma(1-\theta)}, \\ \dfrac{\partial H}{\partial \pi} = 0 \Rightarrow [(1-\pi)y + \gamma c]\lambda = \delta(1-\pi)(1-u)y\mu, \\ \dfrac{\partial H}{\partial u} = 0 \Rightarrow (1-\beta)(1-\pi)\lambda = \delta\pi[(2-\beta)u - (1-\beta)]. \end{cases} \quad (10)$$

Log differentiating the first equation of the system (10) we get

$$\frac{\dot{c}}{c} = -\frac{1}{\theta}\frac{\dot{\lambda}}{\lambda} - \frac{\gamma(1-\theta)}{\theta}\frac{\dot{\pi}}{1-\pi}.$$

As expected, we can observe from this relation that the control variable *c* doesn't evolve independently from the control variable π. From the second and the third equations of the system (10) it immediately follows that

$$u = \frac{(1-\beta)(1-\pi)^2 y + \pi(1-\beta)[\gamma c + (1-\pi)y]}{(1-\beta)(1-\pi)^2 y + \pi(2-\beta)[\gamma c + (1-\pi)y]}, \qquad u \in (0,1).$$

$$\frac{\partial H}{\partial h} = \lambda\beta(1-\pi)\frac{y}{k} + \delta\beta\pi(1-u)\frac{y}{k}\mu,$$

and we can determine the following two differential equations describing the trajectories of λ and *c*.

$$\frac{\dot{\lambda}}{\lambda} = \rho - \beta\frac{y}{k} - \frac{\gamma\beta\pi}{1-\pi}\frac{c}{k}, \qquad (11)$$

$$\frac{\dot{c}}{c} + \frac{1-\theta}{\theta}\frac{\dot{\pi}}{1-\pi} = -\frac{\rho}{\theta} + \frac{\beta}{\theta}\frac{y}{k} + \frac{\gamma\beta\pi}{\theta(1-\pi)}\frac{c}{k}, \qquad (12)$$

$$\frac{\partial H}{\partial h} = \lambda(1-\beta)(1-\pi)\frac{y}{h} + \mu\delta\pi(1-\beta)(1-u)\frac{y}{h}$$

$$\frac{\dot{\mu}}{\mu} = \rho - \delta\pi u\frac{y}{h}$$

Under the hypothesis $(2-\beta)u > 1-\beta$, log differentiating the last equation of the system (10) we get

$$\frac{(2-\beta)\dot{u}}{(2-\beta)u - 1 + \beta} + \frac{\dot{\pi}}{1-\pi} = -\beta\frac{y}{k} - \frac{\gamma\beta\pi}{1-\pi}\frac{c}{k} + \delta\pi u\frac{y}{h}.$$

After some algebraic manipulations and denoting by $z = \frac{hu}{k}$ and $\chi = \frac{c}{k}$ we can close the system and write down the final form

$$
\begin{cases}
\dfrac{\dot{k}}{k} = (1-\pi)z^{1-\beta} - \chi, \\
\dfrac{\dot{h}}{h} = \delta\pi u(1-u)z^{-\beta}, \\
\dfrac{\dot{c}}{c} + \dfrac{1-\theta}{\theta}\dfrac{\dot{\pi}}{1-\pi} = -\dfrac{\rho}{\theta} + \dfrac{\beta}{\theta}\dfrac{y}{k} + \dfrac{\gamma\beta\pi}{\theta(1-\pi)}\chi, \\
\dfrac{(2-\beta)\dot{u}}{(2-\beta)u-(1-\beta)} + \dfrac{\dot{\pi}}{1-\pi} = -\beta z^{1-\beta} - \dfrac{\gamma\beta\pi}{1-\pi}\chi + \delta\pi u^2 z^{-\beta}, \\
\dfrac{\dot{\lambda}}{\lambda} = \rho - \beta\dfrac{y}{k} - \dfrac{\gamma\beta\pi}{1-\pi}\chi, \\
\dfrac{\dot{\mu}}{\mu} = \rho - \delta\pi u^2 z^{-\beta}, \\
u = \dfrac{(1-\beta)(1-\pi)^2 y + \pi(1-\beta)[\gamma c + (1-\pi)y]}{(1-\beta)(1-\pi)^2 y + \pi(2-\beta)[\gamma c + (1-\pi)y]}.
\end{cases} \qquad (13)
$$

4. The balanced growth path

This section is dedicated to analyze the balanced growth path (*BGP* for short), defined as the situation in which the growth rates of per-capita quantities are constant (different from zero) and the growth rates of the share of resources spent on education π, and the fraction of time-labor allocated to the production of physical capital *u*, equal to zero. The following proposition reveals the main result of the paper and examines the properties of the balanced growth path.

Proposition 1 *Let* $\gamma \in (0,1)$ *and* $\theta > 1$. *If there exists a finite* $t_* > 0$, *such that for all* $t \geq t_*$, $r_\pi = r_u = 0$, *then the above system reaches the BGP and the following statements are valid*

1. $r_k = r_c = r_h = r_y = r$, *where* r_x *denotes the growth rate of variable x.*
2. *r is solution of the following nonlinear equation*
$$
\frac{[\delta\rho(1-\beta)]^{1-\beta}\beta^{\beta}}{[\theta+\gamma(\theta-1)]r+\rho(1-\gamma)}\left[\frac{\theta r+\rho}{[(\theta+1)r+\rho]^2}\right]^{1-\beta} = \left[\frac{(\theta+\beta-1)r+\rho}{[\beta+(1+\gamma)(\theta-1)]r+\rho(1+\gamma)}\right]^2. \qquad (14)
$$
3. $$\chi = \frac{c}{k} = \frac{(\theta-1)r+\rho}{\beta}. \qquad (15)$$
4. $$u = \frac{\theta r+\rho}{(\theta+1)r+\rho} \in (0,1). \qquad (16)$$
5. $$\pi = \frac{\theta r+\rho-\beta(r+\chi)}{\theta r+\rho+\gamma\beta\chi}. \qquad (17)$$
6. *r and* χ *are increasing functions of* π, *but u is a decreasing function of* π.

Proof of Proposition 1. From the second equation of the system (13), the hypothesis of constancy of r_h, π and *u*, for all $t \geq t_*$, implies the constancy of z^β from where we get $r_k = r_h$. From the first equation of the system (13), it follows that χ will be also constant and therefore we obtain $r_c = r_k$. Let *r* be the common growth rate of these three variables. Hence we can write

$$\begin{cases} (1-\pi)z^{1-\beta} = r + \chi, \\ \delta\pi u(1-u)z^{-\beta} = r, \\ (1-\pi)z^{1-\beta} + \gamma\pi\chi = \dfrac{1-\pi}{\beta}(\theta r + \rho), \\ \beta z^{1-\beta} = \delta\pi u^2 z^{-\beta} - \dfrac{\gamma\beta\pi}{1-\pi}\chi, \\ u = \dfrac{(1-\beta)(1-\pi)^2 z^{1-\beta} + \pi(1-\beta)\left[\gamma\chi + (1-\pi)z^{1-\beta}\right]}{(1-\beta)(1-\pi)^2 z^{1-\beta} + \pi(2-\beta)\left[\gamma\chi + (1-\pi)z^{1-\beta}\right]}. \end{cases} \tag{18}$$

In order to solve the system above we need the following steps. In the first one we eliminate the variable *z* and thus obtain the system below:

$$\begin{cases} \left[\dfrac{\delta\pi u(1-u)}{r}\right]^{1-\beta} = \left(\dfrac{r+\chi}{1-\pi}\right)^\beta, \\ \dfrac{r+\chi+\gamma\pi\chi}{1-\pi} = \dfrac{ru}{\beta(1-u)}, \\ \pi = \dfrac{\theta r + \rho - \beta(r+\chi)}{\theta r + \rho + \gamma\beta\chi} \Rightarrow 1-\pi = \dfrac{\beta(r+\chi+\gamma\chi)}{\theta r + \rho + \gamma\beta\chi}, \\ u = \dfrac{(1-\beta)(1-\pi)(r+\chi) + \pi(1-\beta)(r+\chi+\gamma\chi)}{(1-\beta)(1-\pi)(r+\chi) + \pi(2-\beta)(r+\chi+\gamma\chi)}. \end{cases} \tag{19}$$

In the second step we eliminate the variable π to obtain the following system

$$\begin{cases} \left[\dfrac{\delta\pi u(1-u)}{r}\right]^{1-\beta} \dfrac{[\theta r + \rho - \beta(r+\chi)]^{1-\beta}}{\theta r + \rho + \gamma\beta\chi} = \left[\dfrac{r+\chi}{\beta(r+\chi+\gamma\chi)}\right]^\beta, \\ [(2-\beta)(\theta r + \rho) - \beta(r+\chi)]u = (1-\beta)(\theta r + \rho), \\ u = \dfrac{\theta r + \rho}{(\theta+1)r + \rho.} \end{cases} \tag{20}$$

In the third step we eliminate the variable *u* and thus we get

$$\begin{cases} \left[\dfrac{\delta\rho(\theta r + \rho)}{[(\theta+1)r+\rho]^2 r}\right]^{1-\beta} \dfrac{[\theta r + \rho - \beta(r+\chi)]^{1-\beta}}{\theta r + \rho + \gamma\beta\chi} = \left[\dfrac{r+\chi}{\beta(r+\chi+\gamma\chi)}\right]^\beta, \\ \chi = \dfrac{(\theta-1)r+\rho}{\beta}. \end{cases} \tag{21}$$

We can now substitute _ into the first equation of the above system and thus we finally obtain the following nonlinear equation

$$\frac{[\delta\rho(1-\beta)]^{1-\beta}\beta^\beta}{[\theta + \gamma(\theta-1)]r + \rho(1-\gamma)} \left[\frac{\theta r + \rho}{[(\theta+1)r+\rho]^2}\right]^{1-\beta} = \left[\frac{(\theta+\beta-1)r+\rho}{[\beta + (1+\gamma)(\theta-1)]r + \rho(1+\gamma)}\right]^2$$

that can be solved only by means of a numerical procedure to determine *r*. Let us now denote by

$$F(r) = \frac{[\delta\rho(1-\beta)]^{1-\beta}\beta^{\beta}}{[\theta+\gamma(\theta-1)]r+\rho(1-\gamma)}\left[\frac{\theta r+\rho}{[(\theta+1)r+\rho]^2}\right]^{1-\beta} - \left[\frac{(\theta+\beta-1)r+\rho}{[\beta+(1+\gamma)(\theta-1)]r+\rho(1+\gamma)}\right]^2 \qquad (22)$$

The derivative of the function *F* with respect to *r* is strictly negative and therefore the function *F* is strictly decreasing. Consequently because

$$F(0) > 0 \; and \; \lim_{r\to\infty} F(r) < 0,$$

there exists a unique positive *r* that satisfies the equation *F*(*r*) = 0. We have now to prove that $\pi \in (0,1)$. Because $\beta(r+\chi) > 0$, what we need is to prove that $\theta r+\rho-\beta(r+\chi) > 0$. We can write

$$\theta r+\rho-\beta(r+\chi) = (\theta-\beta)r-\beta\chi+\rho = (\theta-\beta)r-(\theta-1)r = (1-\beta)r > 0$$

for all *r* > 0 and therefore $\pi \in (0,1)$.
Substituting χ from Eq. (21) into the third Eq. of (19) we obtain

$$r = \frac{\rho(1+\gamma)\pi}{1-(\theta+\gamma\theta-\gamma)\pi} \qquad (23)$$

and the derivative of *r* with respect to π is then given by

$$\frac{dr}{d\pi} = \frac{\rho(1+\gamma)[1+(\theta+\gamma\theta-\gamma)(1-\pi)]}{[1-(\theta+\gamma\theta-\gamma)(1-\pi)]^2} > 0.$$

Taking the derivative of *u* with respect to π into the Eq. (16) we get

$$\frac{du}{d\pi} = \frac{du}{dr}\frac{dr}{d\pi} = -\frac{\rho}{[(\theta+1)r+\rho]^2}\frac{dr}{d\pi} < 0.$$

Taking now the derivative of χ with respect to π into the Eq. (15) we get

$$\frac{d\chi}{d\pi} = \frac{d\chi}{dr}\frac{dr}{d\pi} = \frac{\theta-1}{\beta}\frac{dr}{d\pi} > 0.$$

We have now to check whether the steady state found above, satisfies the transversality conditions. For the two transversality conditions, given by Eqs. (8) and (9), we have that

$$\lim_{t\to\infty}\left[\frac{\dot{k}}{k}+\frac{\dot{\lambda}}{\lambda}-\rho\right] = \lim_{t\to\infty}\left[\frac{\dot{h}}{h}+\frac{\dot{\mu}}{\mu}-\rho\right] = -(\theta-1)r-\rho < 0,$$

and thus the proof is completed.
As we can observe from Eqs. (17) and (23), the variables π and *r* and consequently also *u* and χ, are affected by the level of the constant parameter γ. Numerical simulations show that along the steady state equilibrium, the variables *r*, χ and π are decreasing functions of γ and *u* is an increasing function of γ. Unfortunately, I am unable to prove this assertion, and hope that this open problem could be solved by future researches.

Finally I present the results of a numerical simulation procedure. The benchmark values for the economy we consider are the following: β = 0.25, δ = 0.06, ρ = 0.04, γ = 0.10 ÷ 0.15, θ = 1.5 and coincide with those estimated by Lucas (1988) or considered later by Benhabib and Perli (1994), and Caballe and Santos (1993). The results are presented in the next table.

Table 1. Numerical simulation

γ	r(%)	π(%)	u(%)	$\chi = \frac{c}{k}$	$\psi = \frac{h}{k}$	$z = \frac{y}{k}$
0.10	0.54	7.77	89.86	0.1708	0.2127	0.1911
0.11	0.53	7.61	89.97	0.1707	0.2118	0.1906
0.12	0.53	7.46	90.08	0.1706	0.2109	0.1900
0.13	0.52	7.31	90.19	0.1704	0.2101	0.1894
0.14	0.51	7.16	90.30	0.1702	0.2092	0.1889
0.15	0.50	7.02	90.40	0.1701	0.2083	0.1884

5. Conclusions

Before giving some comments and conclusions onto the numerical simulations, some remarks are necessary. The first part of the paper tries to argue why a different approach is necessary in order to understand better the impact of investment in education on economic growth. Differently to the paper of Chilarescu and Viasu (2014), this paper introduces a new utility function and a new control variable. The two properties of the utility function, non separability and concavity in both variables, are crucial properties that enable us to obtain these results. Of course, the results obtained are quite different from the previous results and this is not at all surprising. In this paper, the share of resources spent on education is a result of the optimization problem, not a fixed quantity as in the cited paper. Another result is also worthy to be pointed out. The share of resources spent on education, determined from the optimal problem, coincides almost exactly with that of developed countries. This is not at all a surprising result. More than this, it is just a confirmation of the validity of the two new hypotheses of the model.

Consequently, a general conclusion is almost obvious. It was proved in this paper, considering the part of resources allocated to the education as a control variable, that this one will increase the skills of human capital and turns out to have significant and positive effects on the long-run economic growth. This evidence goes in favor of a positive growth effect of investment in education that has been questioned in the literature since the contribution of Benhabib and Spiegel (1994).

The results of the numerical simulations are fully consistent with the above theoretical assertions and reinforce earlier results cited in our paper. Namely, investments in educations are the number one investment priority in the developed countries, and our results confirm that this level is close to 7.5% of *GDP*, and it almost coincides with those of developed countries.

One of the main limits of this paper consists in the fact that the solutions obtained are not analytical solutions. Also, the equation that enables us to determine the optimal level of variables can be solved only by numerical simulations. In a future paper we will try to introduce a new utility function and a new equation describing the trajectory of human capital and thus, this approach will improve substantially the findings obtained in this paper.

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