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MODELS FOR OPTIMIZATION OF TRAINING THE UNEMPLOYED

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ABSTRACT. Active Labor Market Policy - including measures such as professional training, job search assistance, wage subsidies on the private sector etc. - belongs to Public Policy which aims to prevent and reduce unemployment. Keeping in mind the negative consequences of the unemployment and the importance for each individual to be able to work, to have a job, we should notice that one of the major problems faced by the national institutions from our country and abroad is the lack of an optimal assignment of the unemployed to professional training programs while taking into account the labor market demand. Therefore, different economic problems concerning the assignment of the persons to attend a professional training program under the restriction of a specified labor market demand can be found in real life situations. The present paper consists of identifying and analyzing two such types of problems that arise and which are solved in practice intuitively.

Key words: *labor force, unemployment, skills, Active Employment Policy*

JEL Classification: J24, J64, J68

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1. Introduction

Starting from the premise that the development of a country, the improvement of people's material and moral well-being, their freedom and happiness depends on work, the problem of employment and unemployment constitutes one of the major interests of macroeconomics. Achieving a high level of employment needs to be one of the priority objectives of the governments' and labor unions' policies of any country. The problem of labor force employment is extremely difficult if we have to take into consideration the fact that each individual has to be employed, in order to be able to ensure the vital elements for his/her private and socio-professional life.

Unemployment is nowadays one of the malfunctions that affect, to different extent, all countries. A high level of unemployment represents both an economical and a social problem. From the economics perspective, unemployment is a waste of precious resources, leading to the diminishing of production and incomes. From a social point of view, it is the cause of deep suffering, regarding the fact that the unemployed struggle to survive despite their low incomes.

The Great Recession started in the US but soon it affected the entire world economy. In Figure 1, we present the percentage change in the unemployment rate in EU countries, Japan and US during the time span 2007-2011.

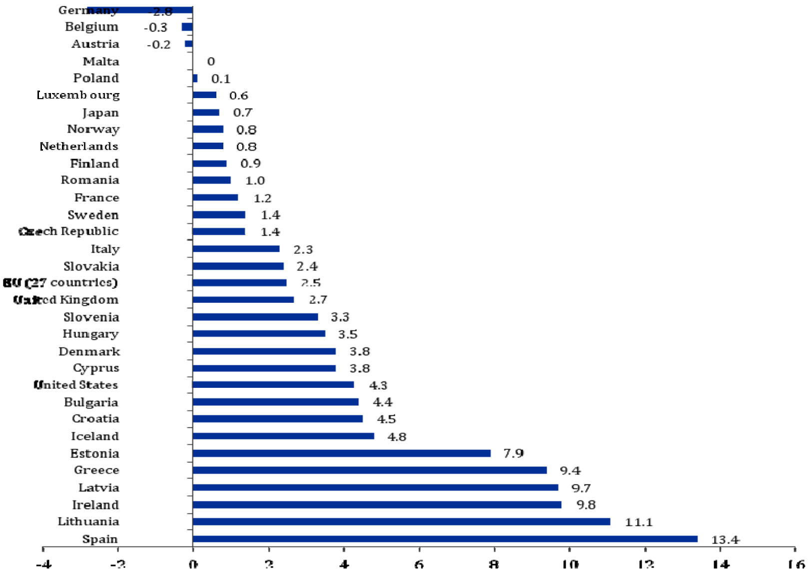

Figure 1. Change in unemployment rate between 2007-2011 (%)
Source: Eurostat, http://epp.eurostat.ec.europa.eu.

We may observe (Figure 1) that the unemployment rate registered the highest increase in Spain (+13.4 %), Lithuania (+11.1 %), Ireland (+9.8), Greece (+9.4), while in three countries (Germany, Belgium, Austria) the unemployment rate registered a slight decrease. During the time span 2007-2011, in US the unemployment rate increases by 4.3 %. According to Schmitt (2011), one possible reason for these differences in unemployment rate is represented by the size of the negative demand shock which might have varied across these economies. For example, Spain could have suffered a larger negative demand shock than the United States, which in turn experienced a worse demand shock than most of the rest of the countries (Schmitt, 2011, p. 2). Another reason could be the measures (monetary, fiscal, active policy) that each country adopted to tackle unemployment. Nevertheless, Junankar (2011) mentioned that "after a short period many Governments were no longer willing to continue the crisis measures of expansionary fiscal policies, and began to cut back on government expenditures and began to worry more about government budget deficits rather than the state of the labour market" (Junankar, 2011, p. 4).

But when we talk about unemployment, we should take into consideration the long-term unemployment, because it is a consequence of the growth of the unemployment rate. The percentage change in the long-term unemployment rate during the time span 2007-2011 is presented in the Figure 2.

The EU Member States have shown distinct patterns regarding unemployment. In Spain, Ireland and Latvia the long-term unemployment rate has increased more than 7 percentage points over the last four years, while in Germany this rate has decreased by more than 2 percentage points. Long-term unemployment has implications both in what regards the professional integration that becomes more and more difficult as time passed and the antisocial manifestations that this phenomenon may generate. According to Janunkar (2011), the long term unemployed not only lose their skills, they lose motivation, they fall ill, so the human capital begins to depreciate. The most affected population categories are the persons over 45 years, the youngsters and the persons with a low level of education. When the lack of the work place appears at a young age, mostly all careers of those involved are affected, because the probability to continue to remain unemployed increases in the future. For example, in a study conducted by the Economic Council of the Labour

Movement it is mentioned that the young Danish workers, who were jobless for at least 10 months in 1994, 15 years later earned 14 % less - or about $ 10,000 less/year than those who were employed as young adults and their probability to be unemployed was almost double compared to other people (Aspect mentioned in Schuman, 2012).

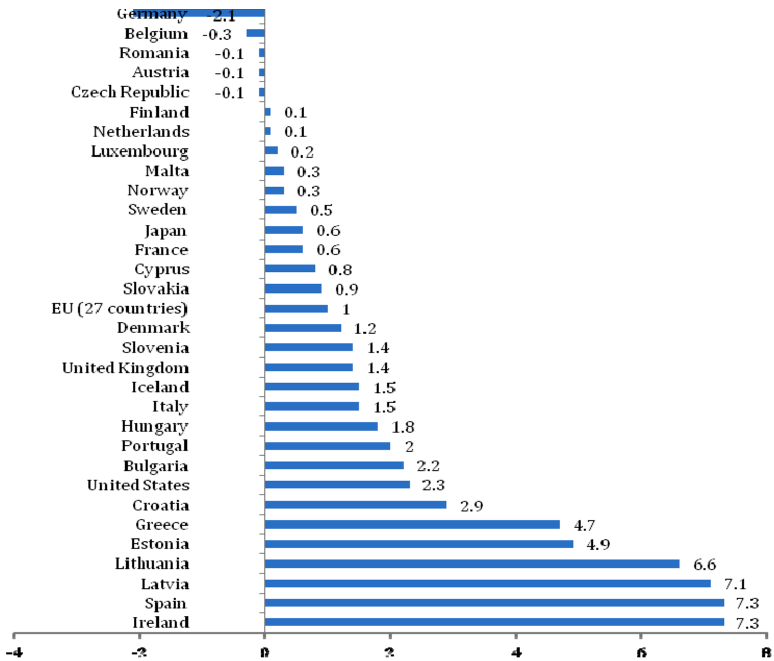

Figure 2. Change in long-term unemployment rate between 2007-2011 (%)
Source: Eurostat, http://epp.eurostat.ec.europa.eu.

During the Great Recession, the youth unemployment rate was almost double (or more) in 2011 compared to 2007 in countries such as: Spain (46.4 % in 20011 - 18.2 in 2007); Lithuania (32.9 % in 20011 -8.2 in 2007); Greece (44.4 % in 20011 -22.9 in 2007); US (17.3 % in 20011 - 10.5 in 2007) (Figure 3).

The high youth unemployment rate can have both economic consequences (the most creative human resources are wasted) and social consequences (the youngster's behavior towards work is negatively affected). Among the factors that affect the youth unemployment rate is the poor correlation that exists between the labor demand and the educational and training system offers. In many countries, the educational system does not prepare the students for the labor market. Sometimes, students choose courses of study that are mismatched with the needs of the economy, because of two reasons: one - because of their personal choice, which is usually not made considering market needs and second - because of the offer of our educational system (Schuman, 2012).

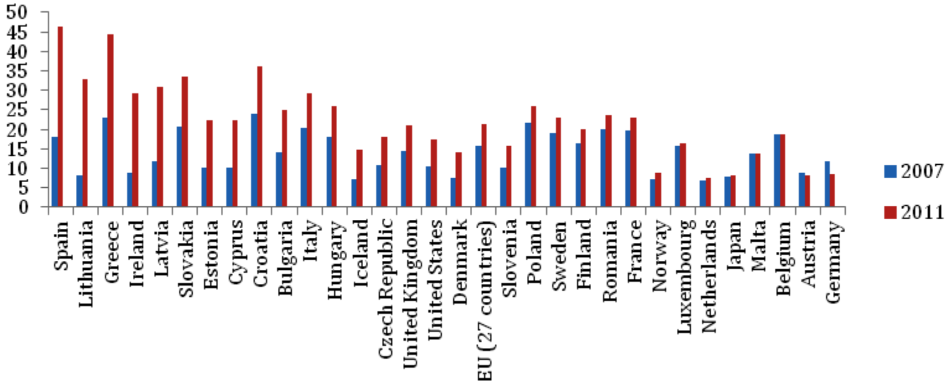

Figure 3. Youth Unemployment Rate in 2007 and 2011 (%)
Source: Eurostat, http://epp.eurostat.ec.europa.eu.

Keeping in mind the negative consequences of the unemployment and the importance for each individual to be able to work and have a job, we should notice the necessity to adopt active measures to help the unemployed and the inactive people to find a work place. The main purpose of the active measures is to prevent and reduce unemployment. But if labor demand is weak and few jobs are available, active policies may be less effective (Nie and Struby, 2011).

In many European countries (Austria, Belgium, Denmark, France, Germany, Italy, Netherlands, Spain, Sweden, United Kingdom etc.) important studies have been conducted regarding the efficiency of the active measures (Zweimuller and Winter-Ebmer, 1992; Lechner, 1999; Winter-Ebmer, 2001; Kluve, Lehmann and Schmidt, 2004; Kluve, 2006; Plesca and Smith, 2006; Nie and Struby, 2011 etc.). The focus of these studies has been on the short and long-term employment effects of the active measures for the unemployed.

As far as *information and professional counseling, and work mediation courses are concerned,* some researchers (Fougere, Pradel and Roger, 2005) consider that these have a *positive impact upon the rate of leaving unemployment,* especially in the case of poorly qualified or low qualification individuals.

According to some authors (Zhang, 2003; Cockx and Gbel, 2005), active measures that aim at *stimulating employment by subvention of work places* contributes to a *faster insertion on the job market of the unemployed* and *to the extension of the duration of employment.*

As far as the *efficiency of professional training courses* is concerned, the opinions vary. Thus some researchers consider that these courses contribute *to the improvement of occupation perspectives* (Winter-Ebmer, 2001; Zhang, 2003; Fitzenberger and Speckesser, 2005; Kluve, Lehmann and Schmidt, 2004), *to work place stability,* (Zweimuller and Winter-Ebmer, 1992), *to the improvement of the quality and efficiency of matching the unemployed (labor supply) with employers (labor demand)* (Nie and Struby, 2011), *to an increase of employment probabilities and incomes* (Heckman, Lanonde, Smith, 1999; Lechner, Miguel and Wunsch, 2005). Others consider that these courses *extend the duration of unemployment,* especially with men (Weber, Hofer 2003; Bolvig, Jensen and Rosholm 2003) and they *have no positive effects in the first years following attendance* (Lechner, 1999).

The aim of these professional training courses also varies depending on the policy of a country/area. So, if we compare the aim of professional training courses in the USA to the ones in the EU, we find that the professional training courses in the USA are mainly aiming at *an increase of productivity and personal incomes,* while in European countries their target is *to prevent or reduce unemployment among low qualification workers with a view to increase the chances of employment rather than income* (Andren, Andren, 2002). Keeping in mind the importance of well-qualified persons, we should notice that one of the major problems faced by the national institutions from our country and from broad is the lack of an optimal assignment of the unemployed to professional training programs taking into account the labor market demand. Therefore, different economic problems concerning the assignment of the persons to attend a professional training program under the restriction of a specified labor market demand can be found in real life situations. The present paper consists in identifying and analyzing two such types of problems that arise and which are solved in practice intuitively.

2. Material and Methods

For the purpose of this analysis we used two types of problems. Both problems deal with the question how to assign *N* unemployed to *M* professional training courses in the best possible way, i.e. to select suitable persons for different professional training courses taking into account the efficiency of each course (defined from the point of view of finding a job by the unemployed after graduating it) and the score that each unemployed has if he/she attends it (this score was calculated based on historical data about each unemployed person taking into account his/her education or professional experience).

Problem I. In the first problem we assume that in the same period of time there are organized different professional training programs (PTPs) for the unemployed persons. The problem that arises is how to assign the registered unemployed persons to the PTPs, based on each person's score, such that the following restrictions to be fulfilled:

i) all unemployed persons to attend the PTPs (i.e. the case when the maximum number of the persons that can attend the PTPs is bigger than the total number of the registered unemployed persons which need to attend the courses);
ii) each unemployed person to attend exactly one PTP;
iii) the assignment of the unemployed persons to a PTP to be done such that to maximize the minimum score of the assignments;
iv) the efficiency of the PTP for which the minimum score is reached to be as small as possible and to be reached as few times as possible. Let us denote by (AEP_1) this first concrete economic problem.

Problem II. In the second problem we work under the hypothesis that restriction i) of the first problem is not fulfilled, i.e. the case when the maximum number of the persons that can attend the PTPs is smaller than the total number of the registered unemployed persons which need to attend the courses, while the above restrictions ii), iii) and iv) occur.

Let us denote by *(AEP_2)* this second concrete economic problem.

In order to find the optimal solution of each studied problem, we propose an algorithm for solving it, highlighted by different examples.

In what follows, let us denote by:

– *m* the number of the total PTPs identified by the variable *i*, $i \in \{1,..., m\}$. Let I = $\{1,..., m\}$.
– e_i, $i \in I$, the efficiency of the PTP *i*;
– *n*, the total number of the unemployed persons that need to attend the PTPs. Let $J = \{1,..., n\}$.
– a_i, $i \in I$, the maximum number of the persons that can participate to the PTP *i*, $i \in I$,
– r_{ij}, $i \in I$, $j \in J$, the score corresponding to each unemployed person *j* if attends the PTP *i*. Let $R \in \mathcal{M}_{m\times n}(\mathbb{R}_+^*)$ be the matrix which elements represent the scores r_{ii};
– y_{ij}, $i \in I$, $j \in J$, the binary variable having the significance $y_{ij} = 1$ if the unemployed person *j* will participate to the course *i* and $y_{ij} = 0$ otherwise.

3. The Study of the Problem (AEP$_1$)

Within the restrictions of our practical problem the values of the efficiencies of the PTPs does not interfere. It interferes just the arrangement of the efficiency of one PTP in relation to the other PTPs. Therefore, we assume that the arrangement of the PTPs was done in a descending order of their efficiency, i.e. $e_i \geq e_{i+1}, \forall i \in I.$

Let $\mathcal{Y}$ be the set of matrices $Y = [y_{ij}] \in M_{m\times n}(\mathbb{R})$ which fulfill the following conditions:

$$y_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J; \tag{1}$$

$$\sum_{i\in I} y_{ij} = 1, \forall j \in J; \tag{2}$$

$$\sum_{j\in J} y_{ij} \leq a_i, \forall i \in I. \tag{3}$$

Let $f = (f_1, f_2, f_3) : \mathcal{Y} \to \mathbb{R}^3$ be the function given by: $\forall Y \in \mathcal{Y}$,

$$f_1(Y) = \min\left\{r_{ij} \,|\, i \in I,\, j \in J,\, y_{ij} = 1\right\} = \min\left\{r_{ij}y_{ij} \,|\, i \in I,\, j \in J\right\}, \tag{4}$$

$$f_2(Y) = \min\left\{i \in I \,|\, \exists\, j \in J \quad \text{such that} \quad r_{ij}y_{ij} = f_1(Y)\right\}, \tag{5}$$

$$f_3(Y) = \sum_{(i,j)\in I\times J;\, r_{ij}y_{ij}=f_1(Y);\, i\geq f_2(Y)} y_{ij} = card\left\{y_{ij} \,|\, i \in I,\, i \geq f_2(Y),\, j \in J,\, r_{ij}y_{ij} = f_1(Y)\right\} \tag{6}$$

Based on (2) and (3) we get that

$$n = \sum_{j\in J}\Big(\sum_{i\in I} y_{ij}\Big) = \sum_{i\in I}\sum_{j\in J} y_{ij} \le \sum_{i\in I} a_i.$$

Hence, we work under the hypothesis that

$$\sum_{i\in I} a_i \ge n, \tag{7}$$

i.e. the total number of the persons that can attend the PTPs is greater than the total number of the registered unemployed persons which need to attend it. Condition (7) assures that $\mathcal{Y} \neq \emptyset$.

In order to give the mathematical model for the (AEP_1) problem we explain the meaning of the order relation of lexicographic type, denoted by $<_{max - max - min}$:

Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ be two elements from $\mathbb{R}^3$.

We say that $x <_{max - max - min} y$ if and only if one of the following conditions occurs:

i) $x_1 < y_1$;

ii) $x_1 = y_1$ and $x_2 < y_2$;

iii) $x_1 = y_1$, $x_2 = y_2$ and $x_3 > y_3$.

Therefore, our problem can be graphically given by the following problem:

$$(PS) \quad \begin{cases} f(Y) \to lex - \max - \max - \min, \\ Y \in \mathcal{Y}. \end{cases} \tag{8}$$

We reduce the solving of the *(PS)* problem to the solving of the following lexicographic optimization problem:

$$(PM) \quad \begin{cases} \varphi(Y) = \begin{pmatrix} f_1(Y) \\ f_2(Y) \end{pmatrix} \to lex - \max - \max, \\ Y \in \mathcal{Y}. \end{cases} \tag{9}$$

We remark that any optimal solution of the *(PM)* problem is an optimal solution of the *(PS)* problem.

Therefore, we give a technique for solving the problem *(PM).*

The idea of the proposed technique is described in the below algorithm. Its efficiency results from the fact that we pass through the scores matrix R from up to down.

In the end, an example to point out how the technique works is given.

Input

the natural numbers m, n;

the elements of natural vector $a = (a_1, \ldots, a_m)$;

the elements of natural matrix $R = [r_{ij}]$, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$;

Output

ok — $true$ if a solution exists,

$Y = [y_{ij}]$, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$ and $F = (F_1, F_2)$ — the solution

Algorithm

```
ok := false; sw := 0;
for j = 1 to n do
    s_j := 0;
    for i = 1 to m do
        y_ij := −1;
    end for
end for
I := {1, ..., m}; J := {1, ..., n};
while J ≠ ∅ do
    r := min{r_ij | i ∈ I, j ∈ J};
    for i = 1 to m do
        if i ∈ I then
          for j = 1 to n do
            if j ∈ J then
              if r_ij = r then
                s_j := s_j + 1; r_ij := +∞;
                if s_j = m then
                  y_ij := 1;
                  if sw = 0 then
                    F_1 := r_ij; F_2 := i; sw := 1;
                  end if
                  a_i := a_i − y_ij;
                  if a_i < 1 then I := I \ {i};
                  end if
```

```
          J := J \ {j};
        else
          y_ij := 0;
          if s_j = m − 1 then
            s_j := m;
            for k = 1 to m do
              if k ∈ I then
                if y_kj = −1 then
                  y_kj := 1; r_kj := +∞; a_k := a_k − y_kj;
                  if a_k = 0 then I := I \ {k};
                  end if
                end if
              end if
            end for
          end if
        end if
      end if
    end if
  end for
 end if
end for
end while
if I = ∅ then OK := false;
end if
if OK = false then
  output: there is an error in the technique;
else
  output: Y = [y_ij] is the optimal solution of the problem (PM) and F = (F_1, F_2)
is the optimal value of the function φ;
end if
End Algorithm
```

Example. Let m = 4, n = **10,** a_1 = 4, a_2 = 3, a_3 = 2, a_4 = 3 and the matrices

$$R = R^1 = \begin{bmatrix} 2 & 4 & 6 & 8 & 1 & 3 & 5 & 7 & 4 & 8 \\ 3 & 5 & 1 & 2 & 7 & 8 & 9 & 7 & 2 & 7 \\ 8 & 1 & 5 & 4 & 8 & 6 & 2 & 3 & 4 & 6 \\ 1 & 7 & 3 & 8 & 4 & 2 & 1 & 4 & 5 & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Iteration 1. We get $a_1 = 4$, $a_2 = 3$, $a_3 = 2$, $a_4 = 3$,

$$R^2 = \begin{bmatrix} 2 & 4 & 6 & 8 & +\infty & 3 & 5 & 7 & 4 & 8 \\ 3 & 5 & +\infty & 2 & 7 & 8 & 9 & 7 & 2 & 7 \\ 8 & +\infty & 5 & 4 & 8 & 6 & 2 & 3 & 4 & 6 \\ +\infty & 7 & 3 & 8 & 4 & 2 & +\infty & 4 & 5 & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Iteration 2. We get $a_1 = 4$, $a_2 = 3$, $a_3 = 2$, $a_4 = 3$,

$$R^3 = \begin{bmatrix} +\infty & 4 & 6 & 8 & +\infty & 3 & 5 & 7 & 4 & 8 \\ 3 & 5 & +\infty & +\infty & 7 & 8 & 9 & 7 & +\infty & 7 \\ 8 & +\infty & 5 & 4 & 8 & 6 & +\infty & 3 & 4 & 6 \\ +\infty & 7 & 3 & 8 & 4 & +\infty & +\infty & 4 & 5 & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Iteration 3. We get $a_1 = 4$, $a_2 = 3$, $a_3 = 1$, $a_4 = 3$,

$$R^3 = \begin{bmatrix} +\infty & 4 & 6 & 8 & +\infty & +\infty & 5 & 7 & 4 & 8 \\ +\infty & 5 & +\infty & +\infty & 7 & 8 & 9 & 7 & +\infty & 7 \\ +\infty & +\infty & 5 & 4 & 8 & 6 & +\infty & +\infty & 4 & 6 \\ +\infty & 7 & +\infty & 8 & 4 & +\infty & +\infty & 4 & 5 & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Iteration 4. We get $a_1 = 4$, $a_2 = 3$, $a_3 = 1$, $a_4 = 2$,

$$R^4 = \begin{bmatrix} +\infty & +\infty & 6 & 8 & +\infty & +\infty & 5 & 7 & +\infty & 8 \\ +\infty & 5 & +\infty & +\infty & 7 & 8 & 9 & 7 & +\infty & 7 \\ +\infty & +\infty & 5 & +\infty & 8 & 6 & +\infty & +\infty & +\infty & 6 \\ +\infty & 7 & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Iteration 5. We get $a_1 = 3$, $a_2 = 2$, $a_3 = 1$, $a_4 = 1$,

$$R^5 = \begin{bmatrix} +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & 7 & +\infty & 8 \\ +\infty & +\infty & +\infty & +\infty & 7 & 8 & +\infty & 7 & +\infty & 7 \\ +\infty & +\infty & +\infty & +\infty & 8 & 6 & +\infty & +\infty & +\infty & 6 \\ +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Iteration 6. We get $a_1 = 3$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$,

$$R^6 = \begin{bmatrix} +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & 7 & +\infty & 8 \\ +\infty & +\infty & +\infty & +\infty & 7 & +\infty & +\infty & 7 & +\infty & 7 \\ +\infty & +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Iteration 7. We get $a_1 = 2$, $a_2 = 1$, $a_3 = 0$, $a_4 = 1$,

$$R^7 = \begin{bmatrix} +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty & 8 \\ +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & 8 & +\infty & +\infty & +\infty & +\infty & +\infty & 8 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Iteration 8. We get $a_1 = 0$, $a_2 = 1$, $a_3 = 0$, $a_4 = 1$,

$$R^8 = \begin{bmatrix} +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \\ +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty & +\infty \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Therefore, Y is the optimal solution of the problem *(PM).*

4. The Study of the Problem *(AEP₂)*

Furthermore we work under the hypothesis that

$$\sum_{i\in I} a_i < n. \tag{10}$$

Using the notations introduced above, we consider the following lexicographic optimization problem:

$$(PMR) \quad \begin{cases} \varphi_1(Y) = \begin{pmatrix} f_0(Y) \\ f_1(Y) \end{pmatrix} \to lex - \max - \max, \\ \sum_{j\in J} y_{ij} \le a_i, \ \forall i \in I, \\ \sum_{i\in I} y_{ij} \le 1, \ \forall j \in J, \\ y_{ij} \in \{0,1\}, \ \forall i \in I, \ \forall j \in J, \end{cases} \tag{11}$$

where

$$f_0(Y)=\sum_{i\in I}\sum_{j\in J} y_{ij} \text{ and } f_1(Y) \text{ is given by (4).}$$

Let us denote by Ω the set of feasible solutions of the problem (PMR), i.e.

$$\Omega=\left\{Y=[y_{ij}]\in\mathcal{M}_{m\times n}(\{0,1\})\mid \sum_{i\in I} y_{ij}\le 1,\ \forall\, j\in J;\ \sum_{j\in J} y_{ij}\le a_i,\ \forall\, i\in I\right\}. \qquad (12)$$

Based on the above, the solving of the problem (PMR) is reduced to solving the following problem:

$$(PMR_1)\quad \begin{cases} f_1(Y)\to \max,\\ \sum_{j\in J} y_{ij}=a_i,\ \forall\, i\in I,\\ \sum_{i\in I} y_{ij}\le 1,\ \forall\, j\in J,\\ y_{ij}\in\{0,1\},\ \forall\, i\in I,\ \forall\, j\in J. \end{cases} \qquad (13)$$

Now, let $r_{m+1,j} := 1+\max\{r_{ij}\mid i\in I,\ j\in J\}$, $a_{m+1} := n-\sum_{i\in I} a_i$ and

$$\bar{I} := I\cup\{m+1\}.$$

Let us consider the problem:

$$(PMR_2)\quad \begin{cases} \min\left\{r_{ij}y_{ij}\mid i\in \bar{I},\ j\in J\right\}\to \max,\\ \sum_{j\in J} y_{ij}=a_i,\ \forall\, i\in \bar{I},\\ \sum_{i\in \bar{I}} y_{ij}= 1,\ \forall\, j\in J,\\ y_{ij}\in\{0,1\},\ \forall\, i\in \bar{I},\ \forall\, j\in J. \end{cases} \qquad (14)$$

We remark that any optimal solution of the *(PMR_2)* problem is an optimal solution of the *(PMR_1)* problem. The solving of the problem *(PMR_2)* can be done applying the technique described in the above paragraph. We just give an easiest example to point out how the proposed technique works in this case.

Example. Let $m = 4$, $n = 10$, $a_1 = 3$, $a_2 = 2$, $a_3 = 1$, $a_4 = 2$ and the matrices

$$R = R^1 = \begin{bmatrix} 2 & 4 & 6 & 8 & 1 & 3 & 5 & 7 & 4 & 8 \\ 3 & 5 & 1 & 2 & 7 & 8 & 9 & 7 & 2 & 7 \\ 8 & 1 & 5 & 4 & 8 & 6 & 2 & 3 & 4 & 6 \\ 1 & 7 & 3 & 8 & 4 & 2 & 1 & 4 & 5 & 8 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We have that

$$\bar{Y} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$Y^* = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

5. Conclusions

In the case of the two examples mentioned above, we can consider that each unemployed person follows the course that helps him/her obtain the highest score, or in other words the course which offers the most benefit regarding obtaining a job in the near future. We consider that these models would be very useful if they would be applied by the workforce agencies in our government. This would allow the individuals that benefit from them to improve their skills in a way that minimizes the time needed for searching and finding a new job.

To follow up on this study, we would like to further investigate what the probability of them successfully finding a job based on following the aforementioned models proposed. We would like to compare the results obtained with those that may currently exist, but we presently have no access to, regarding the potential efficiency of implementing our models.

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